International Journal of Mechanical and Production Engineering Research and Development (IJMPERD)
ISSN (P): 2249-6890; ISSN (E): 2249-8001
Vol. 7, Issue 4, Aug 2017, 99-108

PREDICTION OF NATURAL FREQUENCIES OF THICK CIRCULAR PLATES USING FINITE ELEMENT ANALYSIS

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ABSTRACT

This paper presents linear free vibration of thick isotropic circular plates using finite element simulation. The three-dimensional 'SOLID185' element of ANSYS is used to calculate the natural frequencies of uniform circular plates. Some numerical results are given for various aspect ratios and boundary conditions at outer edges of the plate. The accuracy and convergence of the method are demonstrated by comparison studies with the available solutions in the literature.

KEYWORDS: Thick Plate, Free Vibration, Finite Element & Three-Dimensional Analysis

Received: Jun 19, 2017; **Accepted**: Jul 07, 2017; **Published**: Jul 12, 2017; **Paper Id:** IJMPERDAUG201711

Original Article

INTRODUCTION

Circular plates are commonly used structural component with a wide application in civil, aerospace, mechanical, nuclear and marine engineering. A comprehensive survey of early investigation on free vibration of circular plates made by Leisssa (1969) and Liew et al. (1995) indicates that classical thin plate theory and first order shear deformation plate theory were mainly used by researchers. The classical plate theory neglects the effects of transverse shear deformation and rotary inertia, leading to overestimation of the vibration frequencies. The error increases with increasing plate thickness. To refine classical plate theory by including the shear effect in the analysis of thick plates, various shear deformation and higher-order theories have been proposed during the past few decades.

Rao and Prasad (1975) presented the natural vibrations of annular plates considering the effects of rotatory inertia and shear deformation. Irie et al. (1982) investigated the free vibration behavior of annular plates using Bessel functions. Also, Han and Liew (1999), Liew et al. (2000), Wu et al. (2002), and Wang (2004) applied differential quadrature method to calculate the natural frequencies of circular, annular and sector plates. Hosseini Hashemi et al. (2010) studied the thick circular plates using a third-order shear deformation theory. Due to the physical complicacy and mathematical difficulty, the works concerning three-dimensional theory are comparatively few in the literature. So and Leissa (1998), Zhou et al. (2003), kang and Liessa (1998), Liew and Yang (1999, 2000) and Kang (2003) presented three-dimensional analyses of circular and annular plates using Ritz method.

The three-dimensional analyses of circular and annular plates resting on Pasternak elastic foundation and Winkler foundation investigated by Liew et al. (1996), Zhou et al. (2006), and Hosseini Hashemi et al. (2008). Houmat (2004) investigated the free vibration of annular sector plates using finite element method. Hassani et al. (2010) investigated the in-plane vibration characteristics of annular, circular and elliptic plates of non-uniform

thickness. Liang et al. (2007) employed three node annular finite elements to investigate the natural vibrations of polar orthotropic circular/annular plates of variable thickness. The lateral vibration of thin circular and annular plates was investigated by Chen and Ren (1998) and Khare and Mittal (2015) using finite element analysis. Komur et al. (2010) presented the buckling behavior of laminated composite plate using finite element software ANSYS. Recently, Khare and Mittal (2016, 2017) presented model characteristics of thick laminated circular and annular plates using finite element analysis. In this paper, the free vibration responses of circular plates, with different thickness ratio and boundary conditions, using finite element method are discuss in detail. The accuracy and numerical reliability of method have been verified by, appropriate convergence studies and comparisons of existing results in the literature.

MATERIALS AND METHODS

Consider a homogeneous, isotropic, thick annular plate with inner radius R_1 and outer radius R_2 and uniform thickness h as shown in Figure 1. An orthogonal cylindrical coordinate (r, θ, z) with origin O at the center of the lower surface of the plate, is defined with r in the radial direction, θ in the circumferential direction and z in the thickness direction. The corresponding displacement components at a generic point are u, v and w in the r, θ and z directions, respectively.

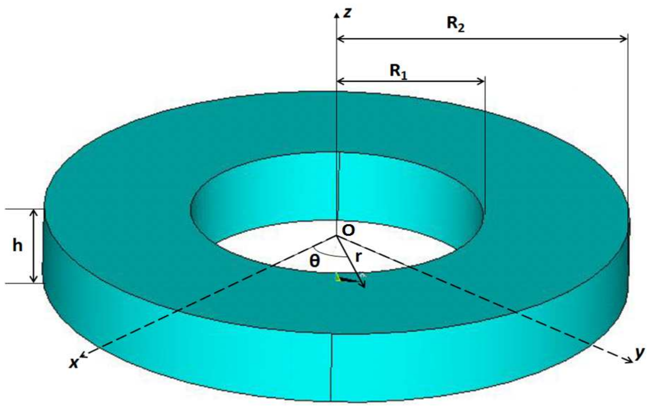

Figure 1: Geometry and Coordinate System of the Annular Plate

A finite element analysis was made for obtaining the first five natural frequencies using three-dimensional 'SOLID185' of ANSYS. The free vibration is computed using Block-Lanczos algorithm. In addition, SOLID185 Structural Solid is suitable for modeling general 3-D solid structures. As demonstrated in Figure 2, the element contains eight nodes having three degrees of freedom at each node: translations in the nodal x, y, and z directions (*ANSYS*, 2009). Figure 3 shows the circular plate subjected to different boundary conditions. The following three types of boundary conditions are considered in the present study.

Clamped boundary condition:

UX = 0, UY = 0, UZ= 0, ROTX = 0, ROTY = 0, ROTZ = 0.

Simply supported boundary condition:

UX = 0, UY = 0, UZ= 0.

Free boundary condition:

UX = UY = UZ= ROTX = ROTY = ROTZ ≠0.

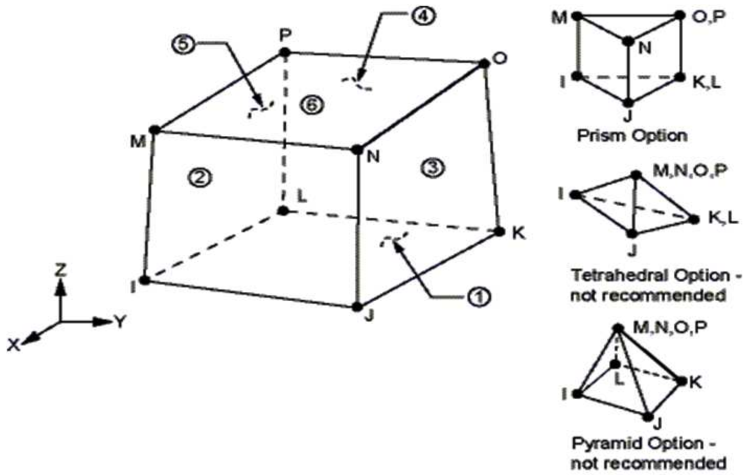

Figure 2: Eight Noded SOLID185 Element

Case 1: Clamp Plate **Case 2: Simply Supported Plate** **Case 3: Free Plate**

Figure 3: The Boundary Conditions of the Circular Plates Analyzed

In this study, isotropic plates made of steel were used. The mechanical properties of steel are listed in Table 1.

Table 1: Mechanical Properties of the Steel (Reddy, 2004)

E_1(GPa)	E_2(GPa)	G_{12}(GPa)	G_{13}(GPa)	G_{23}(GPa)	M
206.85	206.85	77.49	77.49	77.49	0.29

RESULTS AND DISCUSSION

The present study is first validated by carrying out convergence study of non-dimensional frequency parameters Ω defined by $\Omega = \omega R_2^2 \sqrt{\rho h / D}$, with respect to number of element divisions (N) and by comparison with results available in the literature. The rate of convergence of the first five frequency parameters for free, simply supported and clamped boundary conditions is presented in Table 2-4. For free, simply supported boundary conditions, It can be seen that N = 13, is sufficient for converged results, while in case of clamped boundary condition N = 15, is sufficient to obtain satisfactory convergence for first five frequency parameters.

The comparison with result of Irie et al. (1982) shows a very close agreement. Also, the convergence behavior of the frequency parameters for the circular plates with different boundary conditions, are shown in Figure 4-6. The frequency parameters with the superscript symbol∗, represent symmetric thickness mode of vibration. It is found that these modes are more frequent at higher thickness-radius ratios.

The comparison studies are also carried out here in Table 5-7, for circular plate with different boundary conditions, in order to examine the discrepancies between the present finite element solution and Mindlin plate solution,

and those between the present finite element solution and HSDT solution. The comparisons show good agreement with most of differences being less than 4%.

On the basis of above verification of the current approach, frequency parameters for uniform circular plates with thickness-radius ratios varying from 0.1 to 0.5 in step of 0.5 under different boundary condition are presented in Table 8-10. It is observed that value of frequency parameters decreased as the thickness-radius ratios increased. The symmetric thickness modes are frequently occurs at lower frequency spectrum, as thickness-radius ratio increases.

Table 2: Convergence of the First Five Frequency Parameters $\Omega = \omega R_2^2 \sqrt{\rho h / D}$ for Circular Plate with Free Boundary Condition (Relative Thickness h/R_2 = 0.3)

N	Mode Number				
	1(2,0)	2(0,1)	3(3,0)	4(2,0)*	5(1,1)
7	4.9558	8.1592	10.7408	15.9960	16.6897
9	4.9285	8.1022	10.6580	15.9490	16.5280
11	4.9250	8.0971	10.6293	15.9372	16.5278
13	4.8921	8.0341	10.5090	15.9029	16.2577
15	4.8888	8.02638	10.4915	15.8985	16.2568
17	4.87735	8.003	10.4411	15.8842	16.1551
Irie et al. (1982)	4.89	8.01	-	-	15.98

Table 3: Convergence of the First Five Frequency Parameters $\Omega = \omega R_2^2 \sqrt{\rho h / D}$ for Circular Plate with Simply Supported Boundary Condition (Relative Thickness h/R_2 = 0.3)

N	Mode Number				
	1(0,0)	2(1,0)	3(2,0)*	4(1,1)*	5(2,0)
7	4.4729	11.9584	16.0505	18.8139	20.0079
9	4.6617	11.8998	16.0046	18.7466	19.9147
11	4.6258	11.7400	15.9444	18.6563	19.5621
13	4.6062	11.6458	15.9129	18.6133	19.2783
15	4.5991	11.6231	15.9014	18.596	19.2253
17	4.5957	11.5982	15.8942	18.5832	19.2124
Irie et al. (1982)	4.60	11.60	-	-	19.18

Table 4: Convergence of the First Five Frequency Parameters $\Omega = \omega R_2^2 \sqrt{\rho h / D}$ for Circular Plate with Clamped Boundary Condition (Relative Thickness h/R_2 = 0.3)

N	Mode Number				
	1(0,0)	2(1,0)	3(1,0)*	4(2,0)	5(0,1)
7	8.6984	16.0523	22.8229	23.7183	27.0007
9	8.6738	15.9555	22.7642	23.6418	27.17614
11	8.5511	15.7110	22.6452	23.1393	26.2463
13	8.4871	15.5522	22.5864	22.8576	25.6635
15	8.4672	15.5135	22.5649	22.7934	25.5068
17	8.4557	15.4683	22.5520	22.7798	25.4340
Irie et al. (1982)	8.36	15.26	-	22.38	24.64

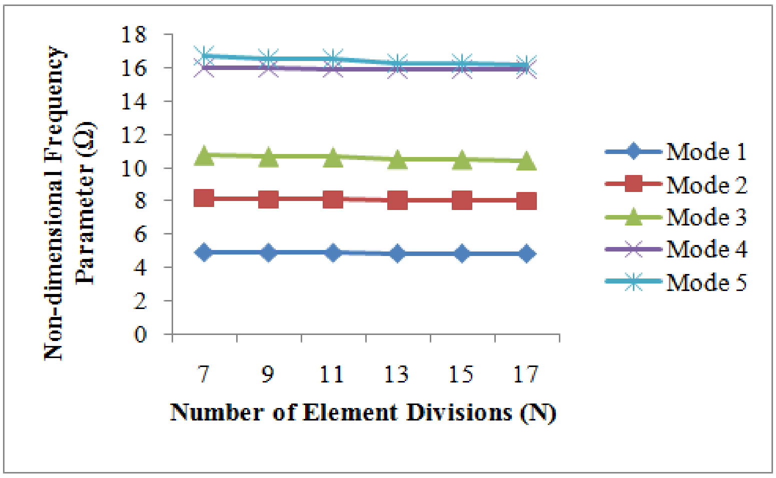

Figure 4: Convergence Behavior of the Frequency Parameter for the Circular Plate with Free Boundary Condition ($h/R_2 = 0.30$)

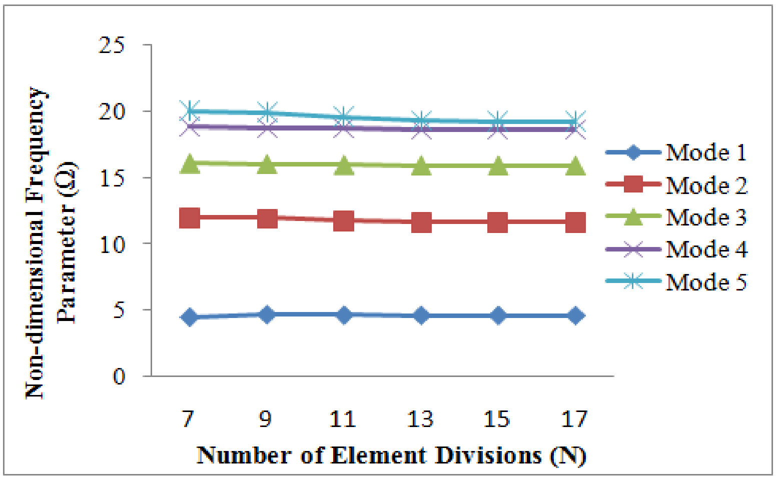

Figure 5: Convergence Behavior of the Frequency Parameter for the Circular Plate with Simply Supported Boundary Condition ($h/R_2 = 0.30$)

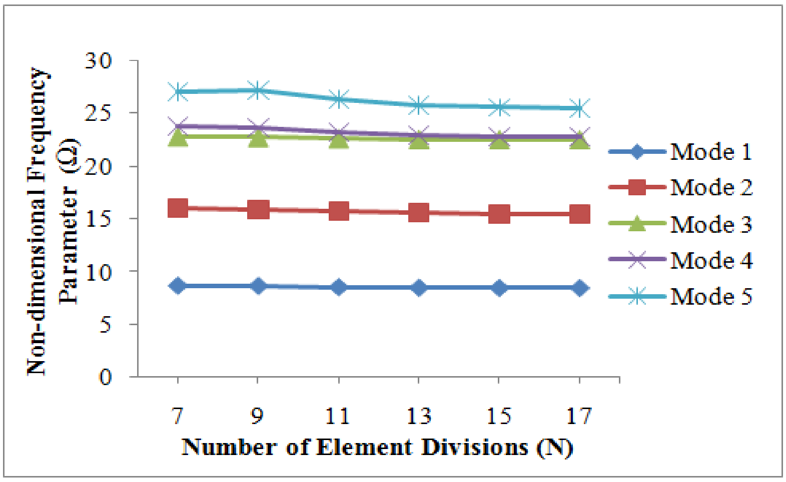

Figure 6: Convergence Behavior of the Frequency Parameter for the Circular Plate with Clamp Boundary Condition ($h/R_2 = 0.30$)

These surface modes are not revealed in 2-D theory based analyses. It can also be observed that value of frequency parameters increases with higher constrained boundary condition at the edge. The first axisymmetric flexural mode (0, 0), is found to be lowest fundamental mode except when the boundaries of the circular plates are free, for which it is found corresponding to mode type (2, 0).

Table 5: Comparison Study of Frequency Parameters $\Omega = \omega R_2^2 \sqrt{\rho h / D}$ for Circular Plate with Free Boundary Condition

h/R_2	Mode Number							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.10	5.2816(2,0)	8.8752(0,1)	12.1475(3,0)	19.9478(1,1)	21.1585(4,0)	32.2031(5,0)	33.8917(2,1)	37.2018(0,2)
#	5.28	8.87	-	19.71	-	-	33.03	36.04
##	5.27842	8.8688	12.0675	19.7172	-	-	33.0529	-
0.15	5.2081(2,0)	8.7314(0,0)	11.8354(3,0)	19.1740(1,0)	20.3049(4,0)	30.4001(5,0)	31.8065(2,0)*	31.8724(2,1)
##	5.20616	8.71147	11.7286	18.9292	-	-	-	30.9798
0.20	5.1178(2,0)	8.5319(0,1)	11.4432(3,0)	18.2799(1,1)	19.2791(4,0)	23.8548(2,0)*	27.8076(1,1)*	28.3320(5,0)
#	5.11	8.51	-	17.98	-	-	-	-
##	5.11607	8.50842	11.3233	17.9983	-	-	-	-
0.25	5.0054(2,0)	8.2849(0,0)	10.9646(3,0)	7.2373(1,0)	18.0878(4,0)	19.0837(2,0)*	22.2416(1,1)*	29.0578(5,0)
##	5.01154	8.27233	10.8788	17.0078	-	-	-	-
0.30	4.8921(2,0)	8.0341(0,1)	10.5090(3,0)	15.9029(2,0)*	16.2577(1,1)	17.0208(4,0)	18.5301(1,1)*	23.3518(0,0)*
#	4.89	8.01	-	-	15.98	-	-	-
##	4.89609	8.01507	10.4176	-	16.0153	-	-	-
0.35	4.5261(0,0)	11.1815(1,0)	13.6724(2,0)*	16.0016(1,1)*	18.1866(2,0)	20.0501(0,0)*	20.9682(0,1)	21.1627(3,0)*
##	4.77299	7.74669	9.9562	--	15.543	-	-	-
0.40	4.6402(2,0)	7.4958(0,1)	9.5761(3,0)	11.9314(2,0)*	13.8900(1,1)*	14.3488(1,1)	14.9984(4,0)	17.4571(0,0)*
#	4.64	7.46		-	-	14.09	-	-

Irie et al. (1982), ## Hosseini Hashemi et al. (2010)

Table 6: Comparison Study of Frequency Parameters $\Omega = \omega R_2^2 \sqrt{\rho h / D}$ for Circular Plate with Simply Supported Boundary Condition

h/R_2	Mode Number							
	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
0.10	4.8951(0,0)	13.6254(1,0)	24.7280(2,0)	29.1557(0,1)	37.9229(3,0)	47.0033(1,1)	47.7325(2,0)*	52.9643(4,0)
#	4.89	13.52	24.41	28.24	-	44.72	-	-
##	4.89421	13.5142	24.3263	28.2547	36.9926	44.7285	-	-
0.15	4.8463(0,0)	13.2838(1,0)	23.6520(2,0)	27.7420(0,1)	31.8632(2,0)*	35.6501(3,0)	37.2560(1,1)*	43.7224(1,1)
##	4.8448	13.1169	23.1045	26.7445	-	34.3771	-	41.2471
0.20	4.7968(0,0)	12.8488(1,0)	22.3471(2,0)	23.9144(2,0)*	26.36975(0,1)	27.9758(1,1)*	32.8608(3,0)	35.2041(0,0)*
#	4.78	12.67	21.92	-	24.99	-	-	-
##	4.77871	12.6324	21.7279	-	25.0414	-	31.6336	-
0.25	4.7160(0,0)	12.3022(1,0)	19.1322(2,0)*	20.8911(2,0)	22.3837(1,1)*	24.4898(0,1)	28.1366(0,0)*	29.6070(3,0)
##	4.69853	12.0979	-	20.325	-	23.319	-	29.0164
0.30	4.6235(0,0)	11.7390(1,0)	15.9509(2,0)*	18.6648(1,1)*	19.4853(2,0)	22.5744(0,1)	23.4274(0,0)*	24.6887(3,0)*
#	4.6	11.6	-	-	19.18	21.59	-	-
##	4.60704	11.5435	-	-	18.9731	21.6757	-	-
0.35	4.5261(0,0)	11.1815(1,0)	13.6724(2,0)*	16.0016(1,1)*	18.1866(2,0)	20.0501(0,0)*	20.9682(0,1)	21.1627(3,0)*
##	4.5069	10.9909	-	-	17.7109	-	20.1569	-
0.40	4.4229(0,0)	10.6398(1,0)	11.9635(2,0)*	14.0035(1,1)*	16.9928(2,0)	17.5121(0,0)*	18.5185(3,0)*	19.5109(0,1)
#	4.4	10.51	-	-	16.74	-	-	18.66

Irie et al. (1982), ## Hosseini Hashemi et al. (2010)

Table 7: Comparison Study of Frequency Parameters $\Omega = \omega R_2^2\sqrt{\rho h / D}$ for Circular Plate with Clamp Boundary Condition

h/R$_2$	Mode Number							
	1	2	3	4	5	6	7	8
0.10	9.9910(0,0)	20.3070(1,0)	32.7823(2,0)	37.3261 (0,1)	46.8313(3,0)	55.6635(1,1)	62.3844(4,0)	67.4498(1,0)*
#	9.94	20.23	32.41	36.48	-	53.89	-	-
##	9.94614	20.1993	32.2634	36.5489	45.8905	53.998	-	-
0.15	9.6836(0,0)	19.2662(1,0)	30.1573(2,0)	34.1821(0,1)	42.1342(3,0)	44.9980(1,0)*	49.4930(1,1)	54.9654(4,0)
##	9.64191	19.1006	29.7933	33.5525	41.4558	-	-	-
0.20	9.3276(0,0)	18.0372(1,0)	27.6318(2,0)	31.1879(0,1)	33.7937(1,0)*	37.8142(3,0)	43.9954(1,1)	48.5018(4,0)
#	9.24	17.83	27.21	30.21	-	-	42.41	-
##	9.26503	17.855	27.2148	30.4749	-	37.1513	42.8755	-
0.25	8.9025(0,0)	16.7463(1,0)	25.0456(2,0)	27.0538(1,0)*	28.1054(0,1)	33.7802(3,0)	33.8035(1,1)	39.0622(2,0)*
##	8.84637	16.5911	24.7887	-	27.6223	33.3357	-	-
0.30	8.4672(0,0)	15.5129(1,0)	22.5656(1,0)*	22.7934(2,0)	25.5068(0,1)	30.3600(3, 0)	34.9711(1,1)	35.1952(2,0)*
#	8.36	15.26	-	22.38	24.64	-	33.47	-
##	8.4113	15.385	-	-	25.1011	30.0729	34.2466	-
0.35	8.0212(0,0)	14.3460(1,0)	19.3500(1,0)*	20.7898(2,0)	23.1684(0,1)	27.4450(3,0)	30.1651(2,0)*	31.3731(1,1)
##	7.97828	14.2728	-	20.715	22.9183	27.3141	-	30.9574
0.40	7.5883(0,0)	13.2891(1,0)	16.9387(1,0)*	19.0543(2,0)	21.1650(0,1)	24.9862(3,0)	26.3901(2,0)*	27.4469(1,1)*
#	7.47	13.04	-	18.64	20.42	-	-	-

Irie et al. (1982), ## Hosseini Hashemi et al. (2010)

Table 8: Value of Frequency Parameters $\Omega = \omega R_2^2\sqrt{\rho h / D}$ for Circular Plate with Free Boundary Condition

h/R$_2$	Mode Number							
	1	2	3	4	5	6	7	8
0.10	5.2816(2,0)	8.8752(0,1)	12.1475(3,0)	19.9478(1,1)	21.1585(4,0)	32.2031(5,0)	33.8917(2,1)	37.2018(0,2)
0.15	5.2081(2,0)	8.7314(0,1)	11.8354(3,0)	19.1740(1,1)	20.3049(4,0)	30.4001(5,0)	31.8065(2,0)*	31.8724(2,1)
0.20	5.1178(2,0)	8.5319(0,1)	11.4432(3,0)	18.2799(1,1)	19.2791(4,0)	23.8548(2,0)*	27.8076(1,1)*	28.3320(5,0)
0.25	5.0054(2,0)	8.2849(0,1)	10.9646(3,0)	7.2373(1,1)	18.0878(4,0)	19.0837(2,0)*	22.2416(1,1)*	29.0578(5,0)
0.30	4.8921(2,0)	8.0341(0,1)	10.5090(3,0)	15.9029(2,0)*	16.2577(1,1)	17.0208(4,0)	18.5301(1,1)*	23.3518(0,0)*
0.35	4.7738(2,0)	7.7755(0,1)	10.0569(3,0)	13.6365(2,0)*	15.3142(1,1)	15.8802(1,1)*	16.0200(4,0)	19.9891(0,0)*
0.40	4.6402(2,0)	7.4958(0,1)	9.5761(3,0)	11.9314(2,0)*	13.8900(1,1)*	14.3488(1,1)	14.9984(4,0)	17.4571(0,0)*
0.45	4.5168(2,0)	7.2463(0,1)	9.1602(3,0)	10.6125(2,0)*	12.3599(1,1)*	13.5329(1,1)	14.1605(4,0)	15.4889(0,0)*
0.50	4.3946(2,0)	6.9763(0,1)	8.7453(3,0)	9.5505(2,0)*	11.1144(1,1)*	12.6954(1,1)	13.3515(4,0)	13.9041(0,0)*

Table 9: Value of Frequency Parameters $\Omega = \omega R_2^2\sqrt{\rho h / D}$ for Circular Plate with Simply Supported Boundary Condition

h/R$_2$	Mode Number							
	1	2	3	4	5	6	7	8
0.10	4.8951(0,0)	13.6254(1,0)	24.7280(2,0)	29.1557(0,1)	37.9229(3,0)	47.0033(1,1)	47.7325(2,0)*	52.9643(4,0)
0.15	4.8463(0,0)	13.2838(1,0)	23.6520(2,0)	27.7420(0,1)	31.8632(2,0)*	35.6501(3,0)	37.2560(1,1)*	43.7224(1,1)
0.20	4.7968(0,0)	12.8488(1,0)	22.3471(2,0)	23.9144(2,0)*	26.36975(0,1)	27.9758(1,1)*	32.8608(3,0)	35.2041(0,0)*
0.25	4.7160(0,0)	12.3022(1,0)	19.1322(2,0)*	20.8911(2,0)	22.3837(1,1)*	24.4898(0,1)	28.1366(0,0)*	29.6070(3,0)
0.30	4.6235(0,0)	11.7390(1,0)	15.9509(2,0)*	18.6648(1,1)*	19.4853(2,0)	22.5744(0,1)	23.4274(0,0)*	24.6887(3,0)*
0.35	4.5261(0,0)	11.1815(1,0)	13.6724(2,0)*	16.0016(1,1)*	18.1866(2,0)	20.0501(0,0)*	20.9682(0,1)	21.1627(3,0)*
0.40	4.4229(0,0)	10.6398(1,0)	11.9635(2,0)*	14.0035(1,1)*	16.9928(2,0)	17.5121(0,0)*	18.5185(3,0)*	19.5109(0,1)
0.45	4.3158(0,0)	10.1223(1,0)	10.6343(2,0)*	12.4453(1,1)*	15.5334(0,0)*	15.9090(2,0)	16.4618(3,0)*	18.1984(0,1)
0.50	4.2066(0,0)	9.5711(1,0)	9.6329(2,0)*	11.2059(1,1)*	13.9456(0,0)*	14.8166(3,0)*	14.9258(2,0)	17.0194(0,1)

Table 10: Value of Frequency Parameters $\Omega = \omega R_2^2\sqrt{\rho h / D}$ for Circular Plate with Clamp Boundary Condition

h/R$_2$	Mode Number							
	1	2	3	4	5	6	7	8
0.10	9.9910(0,0)	20.3070(1,0)	32.7823(2,0)	37.3261 (0,1)	46.8313(3,0)	55.6635(1,1)	62.3844(4,0)	67.4498(1,0)*
0.15	9.6836(0,0)	19.2662(1,0)	30.1573(2,0)	34.1821(0,1)	42.1342(3,0)	44.9980(1,0)*	49.4930(1,1)	54.9654(4,0)
0.20	9.3276(0,0)	18.0372(1,0)	27.6318(2,0)	31.1879(0,1)	33.7937(1,0)*	37.8142(3,0)	43.9954(1,1)	48.5018(4,0)
0.25	8.9025(0,0)	16.7463(1,0)	25.0456(2,0)	27.0538(1,0)*	28.1054(0,1)	33.7802(3,0)	33.8035(1,1)	39.0622(2,0)*
0.30	8.4672(0,0)	15.5129(1,0)	22.5656(1,0)*	22.7934(2,0)	22.7934(0,1)	30.3600(3, 0)	34.9711(1,1)	35.1952(2,0)*
0.35	8.0212(0,0)	14.3460(1,0)	19.3500(1,0)*	20.7898(2,0)	23.1684(0,1)	27.4450(3,0)	30.1651(2,0)*	31.3731(1,1)
0.40	7.5883(0,0)	13.2891(1,0)	16.9387(1,0)*	19.0543(2,0)	21.1650(0,1)	24.9862(3,0)	26.3901(2,0)*	27.4469(1,1)*
0.45	7.2125(0,0)	12.4336(1,0)	15.092(1,0)*	17.7255(2,0)	19.7494(3,0)	23.1722(0,1)	23.5668(2,0)*	24.5587(1,1)*
0.50	6.8180(0,0)	11.5644(1,0)	13.5870(1,0)*	16.3671(2,0)	18.1396(3,0)	21.1757(0,1)	21.3108(2,0)*	22.0469(1,1)*

CONCLUSIONS

In this paper, finite element method has been employed to solve free vibration of thick, isotropic circular and annular plates. The effects of boundary conditions and thickness-radius ratios on different modes of vibration have been fully investigated. It is found that value of frequency parameters decrease as the thickness-radius ratios increases. The symmetric thickness modes frequently occur at higher thickness-radius ratios. It can also be observed that value of frequency parameters increases with higher constrained boundary condition at the edge. The first axisymmetric flexural mode (0, 0), is found to be lowest fundamental mode except when the boundaries of the circular plates are free, for which it is found corresponding to mode type (2, 0).The numerical results revel that the method is very accurate and can be extended to vibration problems of composite laminated plates which are a subject of investigation nowadays.

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